Introductory Math for Econ and Business

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Sequence

- Short math primer for econ. Geometric Sequences
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- A sequence is a set of things (usually numbers) that are in order
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- 3, 7, 11, 15, 19 …

Geometric Sequence

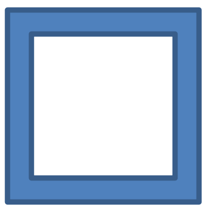
- A geometric Sequence
- A geometric sequence is where the previous number is multiplied by a constant
- 3, 6,12,24,48
- This sequence has a factor of 2 between each number
- This sequence could be written
- = a, ar, ar^2, ar^3
- = {1, 1x2, 1 x 2^2, 1 x 2^3}
- a = 1 = the first term and r = 2 or the common ratio between terms is doubling
- {1 [1], 1 x2 [2], 1 x 2x2 [4], 1 x 2 x 2 x 2 [8], }
- R should not be 0 (zero)

Finding the “nth” term in a Geometric Sequence

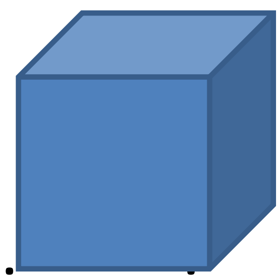
- Let us look at a new sequence:
- 10, 30, 270, 810, 2430
- a = 10 – the first term and r = 3 the “common ratio”
- 10 x 3 = 30, 30 x 3 = 270, etc.
- Adding “…” means continued – open ended means to ∞ infinity. N (n) in this case means the last number in the sequence
- How does one find the value of the nth (1 to N) term?
- What is X4?
- X4 = X^(4-1) = X^3 = 10^3 = 3 x 3 x 3 = 27. 27 x 10 = 270

Why a Geometric Sequence?

- A line is one dimensional and has a length of “r”
- R^2 is two dimensions and is the square r^2 (all sides are supposed to be equal)



- R^3 in 3 dimensional is a cube



… And so on. We can have 4 or more dimensions in mathematics

Summing a Geometric Progression

- Sum means adding together all the elements in the sequence.
- ∑ is the sign sigma. It is used to denote the summing process. Check your XLS column commands (sigma notation)

4

∑ ar^k   Means sum from element 1 to element 4. k is a counter. I is often used as a counter.

N=1 For the sequence 1,2 ,3, 4 the sum (∑)= 10.

- For summing use the equations

  n-1
- ∑(ar^k) = a((1-r^n)/(1-r))

  k=0
- Example – sum the first four terms of : 10, 30, 270, 810, 2430
- 4-1
- ∑(10x3^k) = 10((1-3^4)/(1-3))   = 10 x (1-81)/(1-3) = 10(-80/-2) = 10 x 40 = 400

  k=0

Another Problem

- There are sixty four squares on a chess board. You put one grain of salt on the first square and double it each time. How many grains of salt do you put on the entire board?
- Your input:

  $a = 1$ (the first term)

  $r = 2$ (doubles each time)

  $n = 64$ (number of squares on a chess board)

<u>Solve</u>

1 Using Excel 2. Using your calculator 3. Writing a quick computer program.

Using Excel For these Calculations*

a	r	Adding	N	N - 1
1.00	2.00	1.00	1	
		2	3	2.00
		64	127	7.00
		128	255	8.00
		256	511	9.00
		2.8823E+17	5.76461E+17	59.00
		5.76461E+17	1.15292E+18	60.00
		1.15292E+18	2.30584E+18	61.00
		2.30584E+18	4.61169E+18	62.00
		4.61169E+18	9.22337E+18	63.00
		9.22337E+18	18,446,744,073,709,600,000	64.00

Look our quick this solution is using a simple program such as QBASIC

- a = 1
- r = 2
- n = 64
- sum = a * (1 - (r ^ n)) / (1 - r)
- PRINT sum
- The answer obtained is correct.
- Just make sure you get the parens correct. The program opens the innermost parens first

What if N goes to infinity?

∞

- Let us look at another geometric sequence. This one is inverse. But r is 10 not infinity to show how the sum becomes close to 1.

{1/2,1/4,1/8,1/16…}

a = ½ r = ½ n = 10

10-1

∑(ar^k) = a((1-r^n)/(1-r)) = ½((1-(1/2)^10)/(1-1/2) = ½(1 – 1/1024)/(1/2)

k=0

= 1 – 1/1024 = .9990234375

Work through this or run a simple explanatory program in a simple language like Qbasic

```
a = 1 / 2
r = 1 / 2
n = 10
sum = a * (1 - (r ^ n)) / (1 - r)
Rem This one step gives an accurate answer
P = (1 - (1 / 2) ^ 10)
step1 = 2 ^ 10
PRINT step1;
step2 = (1 / step1)
PRINT step2;
PRINT sum;
REM 1 - sum equals
PRINT 1 - sum;
```

Another Look at the Sum Equation

- Sum ("S") or $\sum$
- $S = a + ar + ar^2 + \ldots + \ldots ar^{n-1}$
- Multiply s by r
- $= Sr = ar + ar^2 + ar^3 \ldots + \ldots + ar^{n-1} + ar^n$

$S = a + ar + ar^2 + \ldots + \ldots ar^{n-1}$

$\underline{-Sr = ar + ar^2 + ar^3 \ldots + \ldots + ar^{n-1} + ar^n}$

$=S – Sr = a$ (all these terms neatly cancel out) $–ar^n$

$S – Sr = a –ar^n$

$S(1-r) = a(1-r^n)$

$S = a(1-r^n)/(1-r)$

Which is our equation

- $\sum(ar^k) = a((1-r^n)/(1-r))$

  $k=0$

QED

Let n go to Infinity∞

- What happens when n goes to infinity and r is between – 1 and plus 1 but not 1 or – 1. If so – we would not get a geometric series. Why?
- 0.01^10000 – try this in excel or in basic -→ goes to 1. Therefore 1 – r = 1.00
- Change to the series {1/2, ¼, 1/8, 1/16,…}
- a = ½ (the first term) and r = ½ (halves each time)

Apply the equation:

$$\sum_{k=0}^{\infty}(ar^k) = \sum(1/2 \times (½^k)) = ½(1 /(1-1/2) = (½ \times 1)/1/2 = 1$$

Adding (1/2) + (1/4) + 1/8) equals exactly 1

Infinity ∞ Continued

- Does .999999’ mean 1? Remember Zeno’s paradoxes? Then, we can’t ignore Max Planck who showed the energy light is not divisible indefinitely. Not for this course? (no worries)
- Calculate 0.999…. (three dots = infinity ∞
- a = 0.9; r = 0.1 Set :
- .999 = 0.9 + 0.9 x 0.1^0 + 0.9 x 0.1^1 + 0.9 x 0,1^2 + ….
- (see spreadsheet on next slide to better understand summing)

∞

= ∑(ar^k) 0.9 x 01^k = 0,9(/1-0.1) = 0.9(1/0.9) = 1

k=0

- Yes 0.9999 does equals 1

Excel

0.999 = 0.9 * .01^0 + 0.9 * 0.01^1 + 09 * 0.01^2 + ….

SUM ∑	Expansion .9* .1^n	Period
0.9000000000	0.9	0
0.9900000000	0.09	1
0.9990000000	0.009	2
0.9999000000	0.0009	3
0.9999900000	0.00009	4
0.9999990000	0.0000090000	5
0.9999999000	0.0000009000	6
0.9999999900	0.0000000900	7
0.9999999990	0.0000000090	8
	0.9999999990	

Perpetual v Long-series Capitalization/Present value Calc.

- How much is a perpetual annuity of $10.00 worth?
- This depends on the subjective discount rates. If your subjective discount rate is 0.05 or 5% - how would you evaluate a perpetual annuity? A quick rule of thumb is to divide the annuity by the discount rate. $10/.05 = $200.
- Use the geometric progression formula for a perpetual annuity.
- a = $10
- r = .05
- Sum of series = 10 x (1/(1-.05)) = $10/.05 = $200. QED
- What is the value of this annuity at a 5% discount for 30 years
- = 10 x(1 - .(05^30))/1-.05) = 1 - .000000?/(1-.05) = close to $200 – this shows that for reasonably long series the simple 1/r x annuity calculation works.